USN | | | | | | | | | | | **10ME64**

Sixth Semester B.E. Degree Examination, Dec.2013/Jan.2014
Finite Element Methods

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

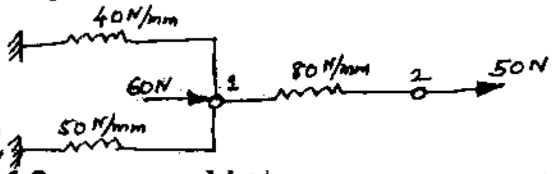
1 a. Differentiate between plane stress and plain strain problems with examples. Write the stress-strain relations for both. **(08 Marks)**

b. Explain the node numbering scheme and its effect on the half band-width. **(06 Marks)**

c. List down the basic steps involved in FEM for stress analysis of elastic solid bodies. **(06 Marks)**

2 a. State the principle of minimum potential energy. Determine the displacements at nodes for the spring system shown in the Fig.Q.2(a). **(08 Marks)**

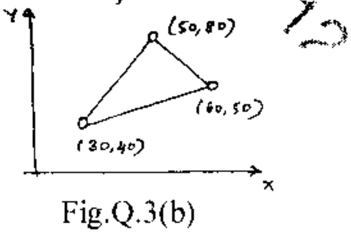
Fig.Q.2(a)



b. Determine the deflection of a cantilever beam of length 'L' subjected to uniformly distributed load (UDL) of P_o/unit length, using the trail function $y = a \sin\left(\frac{\pi x}{2L}\right)$. Compare the results with analytical solution and comment on accuracy. **(12 Marks)**

3 a. Derive an expression for Jacobian matrix for a four-noded quadrilateral element. **(10 Marks)**

b. For the triangular element shown in the Fig.Q.3(b). Obtain the strain-displacement matrix 'B' and determine the strains ϵ_x, ϵ_y and γ_{xy}.
Nodal displacements $\{q\} = \{2 \;\; 1 \;\; 1 \;\; -4 \;\; -3 \;\; 7\} \times 10^{-2}$ mm. **(10 Marks)**



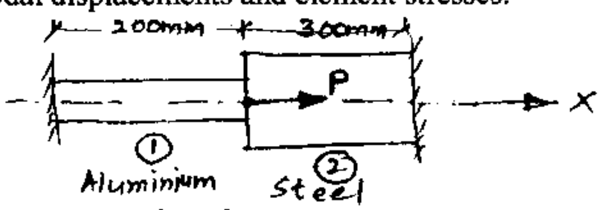
Note: All dimensions in mm. Fig.Q.3(b)

4 a. An axial load $P = 300 \times 10^3$ N is applied at 20°C to the rod as shown in the Fig.Q.4(a). The temperature is then raised to 60°C.

i) Assemble the global stiffness matrix (K) and global load vector (F).

ii) Determine the nodal displacements and element stresses.

Fig.Q.4(a)



$E_1 = 70 \times 10^9$ N/m², $E_2 = 200 \times 10^9$ N/m²
$A_1 = 900$ mm², $A_2 = 1200$ mm²
$\alpha_1 = 23 \times 10^{-6}$/°C, $\alpha_2 = 11.7 \times 10^{-6}$/°C. **(12 Marks)**

b. Solve the following system of equations by Gaussian-Elimination method:
$x_1 - 2x_2 + 6x_3 = 0$
$2x_1 + 2x_2 + 3x_3 = 3$
$-x_1 + 3x_2 = 2.$ **(08 Marks)**

PART – B

5 a. Using Lagrangian method, derive the shape function of a three-noded one-dimension (1D) element [quadratic element]. **(06 Marks)**

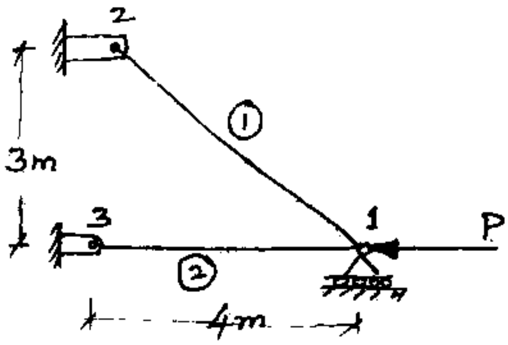
b. Evaluate $I = \int_{-1}^{+1}\left[3e^x + x^2 + \frac{1}{(x+2)}\right].dx$

using one-point and two-point Gauss quadrature. **(06 Marks)**

c. Write short notes on higher order elements used in FEM. **(08 Marks)**

6 a. For the two-bar truss shown in the Fig.Q.6(a). Determine the nodal displacements and element stresses. A force of P = 1000 kN is applied at node 1. Take E = 210 GPa and A = 600mm^2 for each element. **(12 Marks)**

Fig.Q.6(a)

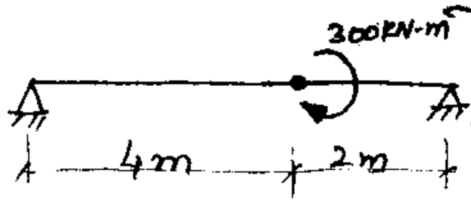


b. Derive an expression for stiffness matrix for a 2-D truss element. **(08 Marks)**

7 a. Derive the Hermite shape functions of a beam element. **(08 Marks)**

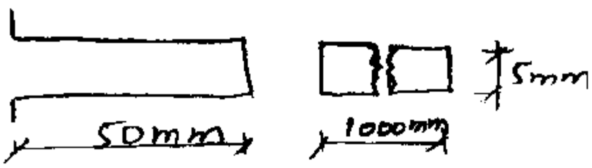
b. A simply supported beam of span 6m and uniform flexural rigidity EI = 40000 kN-m^2 is subjected to clockwise couple of 300 kN-m at a distance of 4m from the left end as shown in the Fig.Q.7(b). Find the deflection at the point of application of the couple and internal loads. **(12 Marks)**

Fig.Q.7(b)



8 a. Find the temperature distribution and heat transfer through an iron fin of thickness 5mm, height 50mm and width 1000mm. The heat transfer coefficient around the fin is 10 W/m^2. K and ambient temperature is 28°C. The base of fin is at 108°C. Take K = 50 W/m.K. Use two elements. **(10 Marks)**

Fig.Q.8(a)



b. Derive element matrices for heat conduction in one-dimensional element using Galerkin's approach. **(10 Marks)**

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